

**QNO1; the diagram shows a sector of a circle with centre  $O$ . The radius of the circle is 8 cm**

**$PRS$  is an arc of the circle**

**$PS$  is a chord of the circle.**

**Angle  $POS = 40^\circ$**

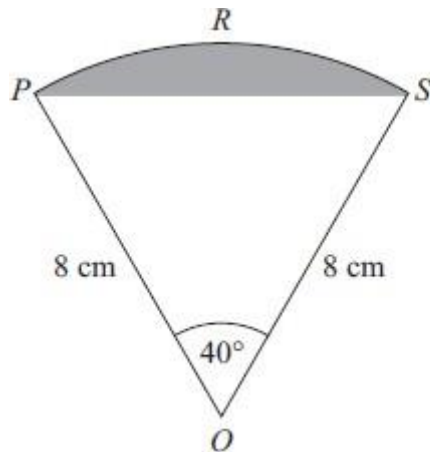


Diagram **NOT**  
accurately drawn

**Calculate the area of the shaded segment.**

**Give your answer correct to 3 significant figures.**

$$\begin{aligned}\text{ANS; } \quad \text{SECTOR} &= \frac{40}{360} \pi (8)^2 \\ &= 22.34021 \text{ cm}^2 \\ \text{Triangle} &= \frac{1}{2} (8)(8) \sin 40^\circ \\ &= 20.56920 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Shaded area} &= \text{sector} - \text{triangle} \\ &= 1.7709 \text{ cm}^2\end{aligned}$$

**QNO 2; in a disastrous first flight, an experimental paper airplane follows the trajectory of the particle**

$$\mathbf{x = t - 3\sin t, \quad y = 4 - 3\cos t \quad (t \geq 0)}$$

**But crashes into a wall at time  $t = 10$**

**(a) At what times was the airplane flying horizontally?**

**(b) At what times was it flying vertically?**

Solution (a). The airplane was flying horizontally at those times when  $dy/dt = 0$  and  $dx/dt \neq 0$ . From the given trajectory we have

$$dy/dt = 3\sin t \text{ and } dx/dt = 1 - 3\cos t$$

Setting  $dy/dt = 0$  yields the equation  $3\sin t = 0$ , or, more simply,  $\sin t = 0$ .

This equation has four solutions in the time interval  $0 \leq t \leq 10$ :  $t = 0$ ,  $t = \pi$ ,  $t = 2\pi$ ,  $t = 3\pi$

Since  $dx/dt = 1 - 3\cos t \neq 0$  for these values of  $t$  (verify), the airplane was flying horizontally at times

$$t = 0, t = \pi \approx 3.14, t = 2\pi \approx 6.28, \text{ and } t = 3\pi \approx 9.42$$

Which is consistent

Solution (b). The airplane was flying vertically at those times when  $dx/dt = 0$  and  $dy/dt \neq 0$ . Setting  $dx/dt = 0$  in yields

$$1 - 3\cos t = 0 \text{ or } \cos t = 1/3$$

This equation has three solutions in the time interval  $0 \leq t \leq 10$

$$t = \cos^{-1} 1/3, t = 2\pi - \cos^{-1} 1/3, t = 2\pi + \cos^{-1} 1/3$$

**Q NO 3; A particle travels along a path defined by the following set of parametric equations. Determine the total distance the particle travels and compare this to the length of the parametric curve itself.**

$$x = 4\sin(1/4t) \quad y = 1 - 2\cos^2(1/4t) - 5/2 \quad \pi \leq t \leq 3/4\pi$$

To find the length we'll need the following two derivatives,

$$dx/dt = \cos(1/4t) \quad dy/dt = \cos(1/4t)\sin(1/4t)$$

The ds for this problem is then,

$$ds = \sqrt{[\cos(1/4t)]^2 + [\cos(1/4t)\sin(1/4t)]^2} dt = \sqrt{\cos^2(1/4t) + \cos^2(1/4t)\sin^2(1/4t)} dt$$

The integral for the length of the curve is now,

$$L = \int_{-\pi}^{3\pi/4} \cos(1/4t) \sqrt{1 + \sin^2(1/4t)} dt$$

$$L = \int_{-\pi}^{3\pi/4} \cos(1/4t) \sqrt{1 + \sin^2(1/4t)} dt$$

$$\sqrt{1 + u^2} = \sec^2 \theta + u^2 = \sec^2 \theta$$

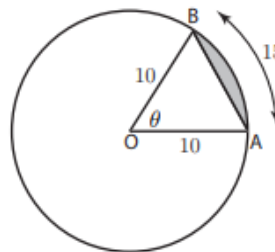
the length of the curve is then,

$$L = \int_{-\pi/2}^{\pi/2} \sqrt{\cos^2(14t)(1+\sin^2(14t))} dt$$

$$= \int_{-1}^1 4\sqrt{1+u^2} du = \int_{\pi/4}^{3\pi/4} 4 \sec^3 \theta d\theta = 2[\sec \theta \tan \theta + \ln|\sec \theta + \tan \theta|] = 9.1824$$

**Qno 4** Refer to Figure 8. Suppose we have a circle of radius 10cm and an arc of length 15cm. suppose we want to find

- (a) The angle  $\theta$ ,
- (b) The area of the sector OAB,
- (c) The area of the minor segment (shaded).



**Figure . The shaded area is called the minor segment.**

- (a) Using  $s = r\theta$  we have  $15 = 10\theta$  and so  $\theta = 15/10 = 1.5$  c.
- (b) Using the formula for the area of the sector,  $A = \frac{1}{2} r^2 \theta$ ,  
We find area  $= \frac{1}{2} r^2 \theta = \frac{1}{2} (10^2) (1.5) = 75 \text{ cm}^2$

We already know that the area of the sector OAB is  $75 \text{ cm}^2$ .

If we can work out the area of the triangle AOB we can then determine the area of the minor segment. (Recall the formulae for the area of triangle,  $A = \frac{1}{2} ab \sin C$ .)

$$\text{Area of triangle} = \frac{1}{2} r^2 \sin \theta = \frac{1}{2} (10^2) \sin 1.5 = 49.87 \text{ cm}^2$$

Therefore the of the minor segment is  $= 75 - 49.87 = 25.13 \text{ cm}^2$  (to 2 dp.)